Polyphase Filter Banks Using Wave Digital Filters

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Abstract—The branch filters in a digital polyphase network can be designed either as FIR filters by decomposing the impulse response of an FIR low-pass prototype filter, or as ordinary IIR filters by the synthesis method of Bellanger. The use of all-pass networks has hitherto been considered unfeasible because of the associated computational difficulties involved in the design of filter banks with many branches.

The purpose of this paper is to demonstrate that it is indeed possible to design the branch filters as all-pass-low-pass sections without the need of a prototype filter. Moreover, these sections can be realized as wave digital filters, which give improved properties over the other designs with respect to hardware requirements, group delay, sensitivity, dynamics and limit cycles. Examples, including the design of the practically important 60-channel filter bank for the transmultiplexer, are given.

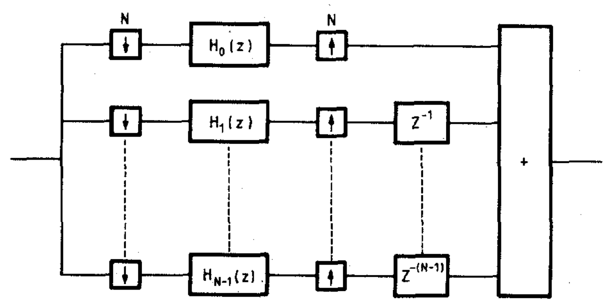

Fig. 1. Low-pass polyphase network. Arrows denote sample rate decrease and increase, respectively.

I. Introduction

IN certain digital signal processing areas such as sample rate alteration and FDM-TDM conversion by the transmultiplexer, a special type of digital low-pass filter based on phase shifting has been used. This is the so-called polyphase network, which was first introduced by Bellanger [1], [2] and further developed by Vary [10]. The sampling frequency of this device is twice the cutoff frequency of the filter, which means that the signal processing is done at the lowest possible rate. Thus, the overall computation rate will be minimized, which is a necessity for the 60-channel FDM-TDM conversion.

The design of the polyphase network has hitherto been based on the special properties of digital signal processing, such as the possibility of directly realizing a transfer function. It is, however, well known that analog filtering techniques are to be preferred in many instances also in digital filtering in order to reduce, for example, sensitivity, which has implications on word length and noise. Unfortunately, the phase shifter cannot be used for demanding analog filtering purposes because of the stringent tolerance requirements on the network components and, as a consequence, this problem has not received much attention (other than for the special case of SSB signal generation). Hence, there is no straightforward way to apply analog filtering techniques in this context.

In this paper we will thus be concerned with the problems of designing a digital polyphase network utilizing the analog filtering concepts developed for digital filtering, and in particular that of wave digital (WD) filters [3]. The main difficulty here is to find a suitable approximation procedure that works in practice for the 60-channel transmultiplexer, which is the most important application in the telecommunication industry. The problem is solved by approximating the phase response of first-order all-pass sections and cascading these sections with suitable low-pass filters. It will be demonstrated by examples that such an approach has definite advantages over the previously published design methods.

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II. Problem Formulation

The polyphase network in Fig. 1 realizes the digital low-pass filter $H(Z)$ given by

$$H(Z) = \sum_{m=0}^{N-1} Z^{-m} H_m(Z^N) \qquad m = 0, 1, \cdots, N-1$$

$$Z = e^{j\theta} \qquad -\pi \leqslant \theta \leqslant \pi, \quad \theta = 2\pi f/f_s$$

$$\theta_c = \pi/N = \text{cutoff frequency} \qquad f_c = f_s/2N \tag{2.1}$$

with f_s = the sampling frequency of $H(Z)$ and N = the number of branches.

Alternatively, we may write (see the Appendix)

$$H_m(z) = z^{m/N} \cdot \frac{1}{N} \sum_{l=0}^{N-1} W_N^{ml} \cdot H(z^{1/N} W_N^l)$$

$$W_N = e^{-j(2\pi/N)}$$

$$z = Z^N = e^{j\omega} \qquad -\pi \leqslant \omega \leqslant \pi, \quad \omega = 2\pi f N/f_s. \tag{2.2}$$

Furthermore, the impulse responses of $H(Z)$ and $H_m(z)$ are connected by

$$h_m(n) = h(nN + m) \tag{2.3}$$

which means that the branch filters $H_m(z)$ are working N times slower than $H(Z)$. Thus, the overall computation rate is kept at a minimum.

When designing the polyphase network, the first step is usually to compute a low-pass FIR or IIR prototype filter $H(Z)$ by standard methods such as the Remez algorithm [7] for FIR filters or the bilinear transformation of an analog reference fil-

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ter for IIR filters. The branch filters $H_m(z)$ are then derived in some manner from the prototype filter.

For the FIR case, we can directly determine $H_m(z)$ from $\{h(n)\}$ by choosing

$$h_m(n) = h(nN + m)$$
$$\text{for } n = 0, 1, \cdots, K, \; K = \text{degree of } H(Z) \quad (2.4)$$

which will yield the desired $H(Z)$. It is, however, difficult to design channel-bank filters for the 60-channel transmultiplexer in this way because $H(Z)$ will have a large number of coefficients (2000-4000), all of which must be determined by the approximation algorithm. Other disadvantages are that more hardware will be needed and the group delay will be greater compared with corresponding IIR filters.

For IIR filters, Bellanger [2] has suggested a synthesis procedure to directly compute $H_m(z)$ from $H(Z)$, giving

$$H_m(z) = P_m(z)/Q(z) \quad (2.5)$$

which means that all filters have the same denominator. The realization is done either in the direct form or, as in the transmultiplexer, by cascading an all-pole recursive filter with a symmetric FIR filter, corresponding to $1/Q(z)$ and $P_m(z)$, respectively. The symmetry is then utilized to reduce the necessary hardware.

Both of these realizations have certain disadvantages. First of all, it is well known that the direct form is very sensitive to changes in filter coefficient values and hence must be realized with high precision. Second, in the transmultiplexer case, the magnitudes of $P_m(z)$ and $Q(z)$ are heavily attenuated (>40 dB) towards the band edge, which means that flat passbands are achieved through compensation. Such a structure has bad internal dynamics (i.e., the signal level varies considerably throughout the filter). Thus, the signal has to be represented by many bits in order to avoid information loss. For the 60-channel transmultiplexer filter, for example, Bellanger uses 16 bits [1]. Finally, since all filters have the same denominator, all degrees of freedom have obviously not been used in the design, and it should be possible to use for example filters of less order if the denominators are allowed to differ.

Hence, it would be desirable to use other structures, and in particular WD filters, which are derived from resistively terminated lossless and passive analog reference filters. It has been verified by numerous contributions that WD filters have very good sensitivity and dynamic properties and, in addition, limit cycles can be suppressed [4].

There are, however, certain difficulties involved with the use of such filters in this context. The analog reference filter might in principle be found by taking the transfer function $H_m(z)$ achieved from the Bellanger method and perform an inverse bilinear transform to get the analog transfer function $H_m(s)$. In order to realize $H_m(s)$ by passive circuits, certain conditions must be fulfilled for the poles and zeros of $H_m(s)$. Ladder filters, for example, require all transmission zeros to be in the left half of the s-plane. It is not very likely that the polyphase network can be made up from such networks only because of the stringent phase requirements. However, if $H_m(s)$ has transmission zeros in the right half-plane, these must have corresponding poles (for all-pass sections) or zeros (for quadruple zero sections) in the left-half plane. It turns out that these conditions are generally not fulfilled by the transformed $H_m(z)$. Moreover, the problem of doing a synthesis (that is, finding the component values) from $H_m(s)$ still remains.

Hence, we would like to find another design procedure for WD filters in this context or, for that matter, any digital filter based on passive reference filters, and in the next section an approximation procedure is given that makes it possible to design $H_m(z)$ as WD all-pass-low-pass sections.

III. Method of Computation

If (2.2) is evaluated along the unit circle, we get

$$H_m(e^{j\omega}) = e^{j\omega(m/N)} \cdot \frac{1}{N} \sum_{l=0}^{N-1} W_N^{ml} \cdot H(e^{j(\omega - 2\pi l/N)}). \quad (3.1)$$

When $H(Z)$ is an ideal filter, i.e.,

$$H(e^{j\theta}) = \begin{cases} 1 & |\theta| \leqslant \pi/N \\ 0 & |\theta| > \pi/N, \end{cases} \quad (3.2)$$

(3.1) becomes

$$H_m(e^{j\omega}) = \frac{1}{N} e^{j\omega(m/N)}. \quad (3.3)$$

If we only require that $H(Z)$ is band-limited to $|\theta| \leqslant \pi/N$, it is sufficient if

$$H_m(e^{j\omega})/H_k(e^{j\omega}) = e^{j\omega(m-k/N)}$$
$$-\pi \leqslant \omega \leqslant \pi, \quad k, m = 0, \cdots, N-1 \quad (3.4)$$

which means that

- the difference in phase response between H_m and H_k shall be linear in ω,
- $|H_m| = |H_k|$.

With all-pass sections, the second condition is automatically fulfilled, and only the phase requirements have to be approximated. However, to our knowledge, there are no explicit methods available for this type of approximation, although some related work has been done in the design of all-pass sections for the quadrature modulator (see, for example, [5]). Thus, we are confined to ordinary optimization methods for the approximation.

This approach has certain difficulties. First of all, we have to determine the order and form (complex or real poles) of the all-pass sections, as well as appropriate starting values for the parameters. It is then, of course, desirable to keep the order of the all-pass sections as low as possible. An investigation shows that sections with real poles in the interval $\mathrm{Re}(z) \in \{-1, 0\}$ are probably the best choice. This means that

$$H_m^{AP}(z) = \prod_{r=1}^{R^{(m)}} \frac{1 - za_r^{(m)}}{z - a_r^{(m)}} \quad (3.5)$$

where $R^{(m)}$ is the number of sections in the mth branch and $a_r^{(m)}$ is the singularity of the rth section in that branch.

Furthermore, since the phase difference between any two digital transfer functions with real poles at $\omega = \pi$ is an integer times π, and the desired phase difference at that frequency is an integer times π/N, it is clear that the approximation cannot be carried all the way to $\omega = \pi$. The effect of this will be attenuation dips in $H(Z)$ around the frequencies

$$\theta = \theta_c + l 2\pi/N \qquad l \text{ integer}, \quad 1 \leqslant l \leqslant N/2.$$

At those frequencies, the magnitude of (2.1) reduces to

$$\left| \sum_{m=0}^{N-1} e^{-j\theta m} \right|$$

when $H_m(z)$ are all-pass networks. This expression decreases as l increases, which means that the attenuation dips will be less toward higher frequencies [see also Fig. 8(a)].

In some instances, these dips will not be severe. This is the case, for example, if the requirements are stated in a weighted integral form as in the CCITT requirements for the channel-bank filter. In general, though, the attenuation must be above a certain level at all frequencies in the stopband.

In such cases, the attenuation dips can be filtered out by suitable low-pass filters in cascade with the all-pass sections. If these low-pass filters are made identical for all branches, the phase difference will not be affected. This also has the advantage that only one low-pass filter has to be designed, and thus more effort can be spent in optimizing that design.

Thus, we will write $H_m(z)$ as

$$H_m(z) = H^{\mathrm{LP}}(z)\, H_m^{\mathrm{AP}}(z). \tag{3.6}$$

It turns out that the LP filter can be designed almost independently from the all-pass sections. For the high stopband attenuations we are interested in (>60 dB), the passband distortion from the all-pass sections is negligible. Furthermore, as can be derived from (2.2), $H_m(z)$ and $H(Z)$ will have approximately the same response in the baseband (that is, $0 \leqslant \theta \leqslant \pi/N$) since the aliasing effects will be very small. Thus, the LP filter can be designed to meet the requirements in this frequency band, while the all-pass sections will take care of the rest of the stopband.

When designing the 60-channel filter bank for the transmultiplexer, another difficulty emerges. This bank has 128 branches and, in principle, all singularities of the filters are designable. Hence, the optimization method must handle a large number of variables (300–500), which is numerically difficult. However, since a corresponding prototype filter $H(Z)$ would have a much smaller number of designable parameters, it should be possible to find a parametric representation of the filter bank variables. In other words, there should be some form of interdependence among those variables. Now the difference in phase between two consecutive filters will be very small for large N (in fact, $< \pi/N$ for $0 \leqslant \omega \leqslant \pi$), which means that the pole positions will change very little. This suggests that it would be possible to select certain filters for the optimization and use these as "pivot" filters. The values of the poles in the other sections are then found by interpolation between the poles of the pivot filters. It has been found that this procedure works well in practice.

Since the optimization is carried out on the phase response in the baseband, we have no direct control of the magnitude response of $H(Z)$ in the whole of the interval $0 \leqslant \theta \leqslant \pi$. To check this would require a certain number (10–20) of frequency points in each basic frequency band of width $2\pi/N$, which means a prohibitive number of points in the 60-channel case since there are 64 such bands. However, due to the properties of (2.1) mentioned earlier, the lowest stopband attenuation of $H(z)$ can be expected in the bands adjacent to the baseband. This makes it unnecessary to check every frequency band during the progress of the optimization, but the final solution, of course, has to be examined more carefully.

It would also in principle be possible to optimize the magnitude response directly with the pivot filter poles as variables. This, however, requires a lot more computational effort, since all 128 filters must be known before $H(Z)$ can be evaluated. It also turns out that the numerical problems are more severe with this procedure.

Finally, a remark about the group delay. For the transmultiplexer, it is important to have a low group delay, especially at low frequencies. In the Appendix it is shown that the group delay for $H(Z)$ is approximately equal to that of $H_o(z)$. Thus, if $H_o(z)$ is designed to have a low group delay (which means, for example, a low order), $H(Z)$ will also have this property.

IV. Examples

Two filters for the transmultiplexer have been calculated with the procedure above. The first one is a low-pass filter for a four-channel transmultiplexer with an overall sampling frequency of 32 kHz. The eight filters in the filter bank work at 4 kHz. For comparison, this bank has also been computed with the Bellanger method in which the all-pole filter $1/Q(z)$ was designed as cascaded second-order sections.

The first step in the design is to find a suitable low-pass filter $H^{\mathrm{LP}}(z)$ by the standard bilinear transformation. One possible choice is a fourth-degree filter with one finite frequency attenuation pole. For the all-pass filters, three first-order sections were found sufficient, i.e., $R^{(m)} = 3$ for all m. It is possible that the high-number channels could do with only two all-pass sections, since they have a smaller phase shift. This, however, has not been investigated. The poles (altogether 24) were then optimized to meet the phase requirements in the range $0 < f < f_P$ where f_P is an adjustable frequency parameter. By changing f_P, a tradeoff between requirements on $H^{\mathrm{LP}}(z)$ and $H_m^{\mathrm{AP}}(z)$ is achieved, since a smaller f_P gives a better phase approximation. On the other hand, a wider interval then has to be covered by $H^{\mathrm{LP}}(z)$. In this case, f_P was 1.85 kHz.

The Fletcher method [6] was used throughout the optimization, and the computations were carried out with an internally developed program using DINAP [8] as a subprogram for the digital circuit analysis.

In Fig. 2, the result of the Bellanger method is shown, and in Fig. 3, that of the WD design. The Bellanger filters are obviously more sensitive to coefficient truncation. Whereas Bel-

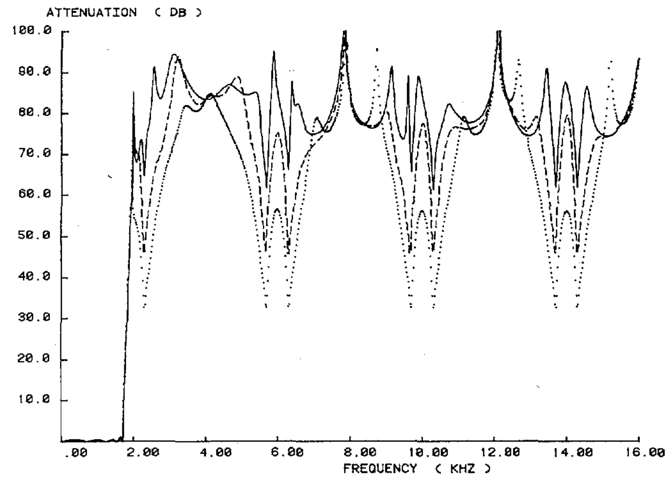

Fig. 2. Response of four-channel transmultiplexer filter designed by the Bellanger method. The coefficients of the filter in one channel are truncated to: —— = 16 bits, – – – = 12 bits, · · · · = 10 bits.

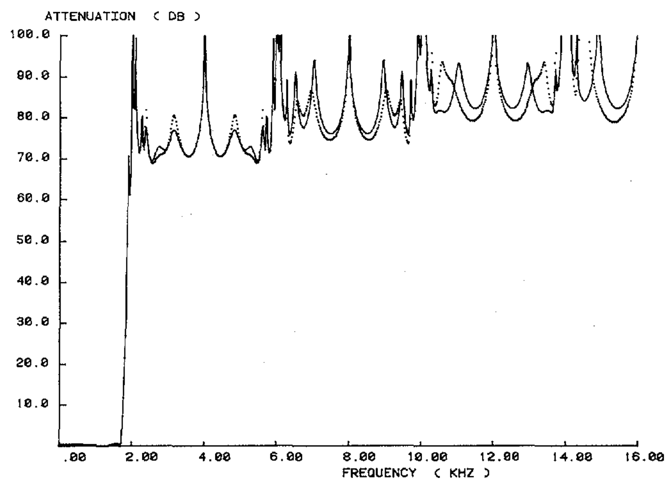

Fig. 3. Response of four-channel transmultiplexer filter with WD all-pass–low-pass sections. The coefficients of the filter in one channel are truncated to: —— = 16 and 12 bits, · · · · = 10 bits.

langer needs 16 bits to hold a 60 dB stopband attenuation, the WD design needs only 8 to 10 bits. Also, the group delay is considerably better for the WD design (Fig. 4).

One possible WD realization is shown in Fig. 5, where the all-pass sections are realized as three-port circulators [3]. The hardware requirement of this solution as compared with that of Bellanger (utilizing the symmetry of the FIR filter) is the following.

	WD		
	LP	AP	Bellanger
Multipliers	48	24	108
Adders	160	72	104
Delays	40	24	108

It can be seen that the WD design needs less hardware and, since most of this is used in the LP filters, the hardware can be further reduced if the low-pass filters are not necessary or can be of a smaller order. It should also be kept in mind that the multipliers of the WD low-pass filter can be realized with shift and add only, due to the low sensitivity of that filter.

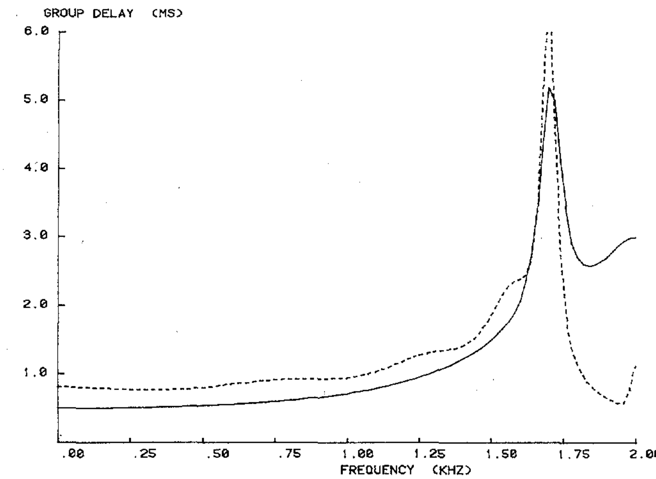

Fig. 4. Group delay of four-channel transmultiplexer filter: —— = WD filter, – – – = Bellanger filter.

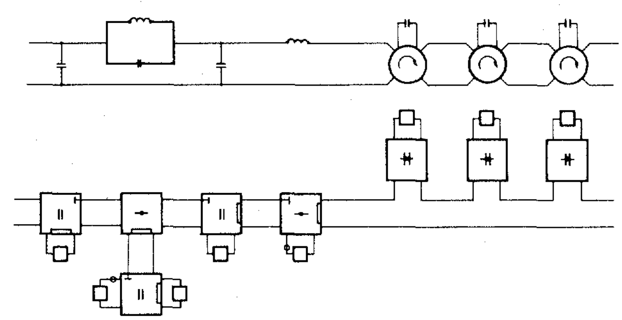

Fig. 5. Realization of a WD low-pass–all-pass branch.

TABLE I
POLE POSITIONS FOR THE PIVOT FILTERS

Pole Positions for the Pivot Filters				
Filter number	Section number			
	1	2	3	4
0	-.8293	-.5066	-.1547	-.003941
18	-.8716	-.5541	-.2014	-.007574
36	-.9066	-.6036	-.2471	-.01795
54	-.9362	-.6510	-.2963	-.03254
73	-.9618	-.6988	-.3501	-.05410
91	-.9819	-.7405	-.4028	-.08071
109	-.9969	-.7808	-.4543	-.1148
127	-.9999	-.8278	-.5010	-.1580

The second example is a similar filter for the 60-channel filter bank. It consists of 128 branch filters with the previous low-pass filter and four all-pass sections in each branch. As pivot filters were chosen filters 0, 18, 36, 54, 73, 91, 109, and 127, and all poles except the one nearest $z = -1$ were taken as variables. The optimization usually converged very quickly (typically in less than 400 function evaluations), and resulted in the pole positions summarized in Table I for f equal to 1.70 kHz. The remaining poles were then interpolated from these values.

The resulting frequency response is given in Fig. 6. As can be seen, the attenuation of this bank is as good or better than that of the four-channel filter bank. This is due to the fact

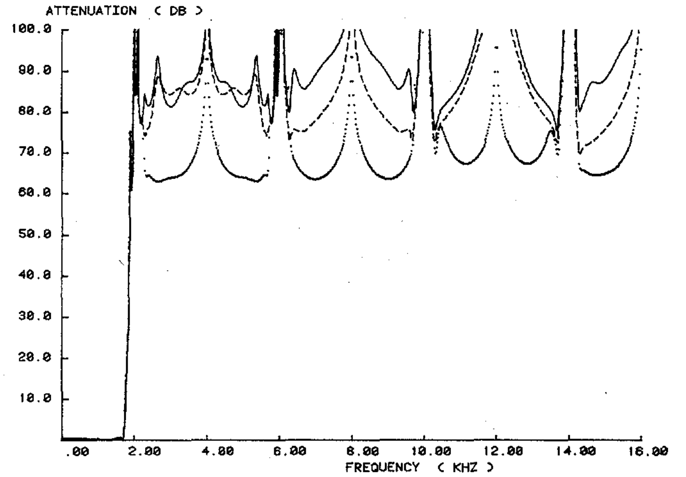

Fig. 6. Response of 60-channel transmultiplexer filter with four WD all-pass sections. All coefficients are truncated to: —— = 12 bits, – – – = 10 bits, · · · · = 8 bits.

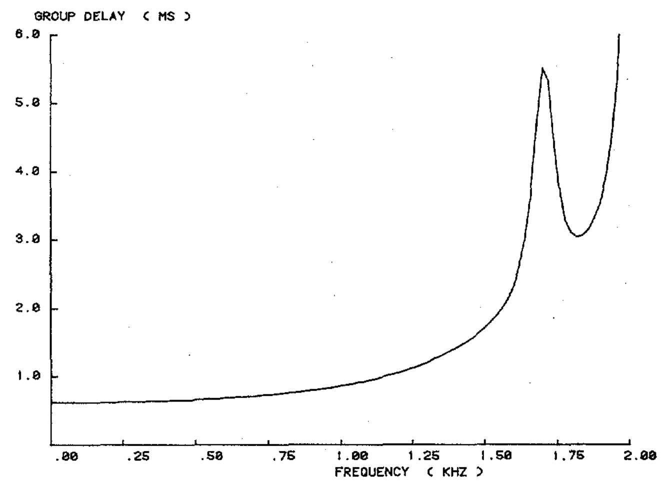

Fig. 7. Group delay of 60-channel transmultiplexer filter.

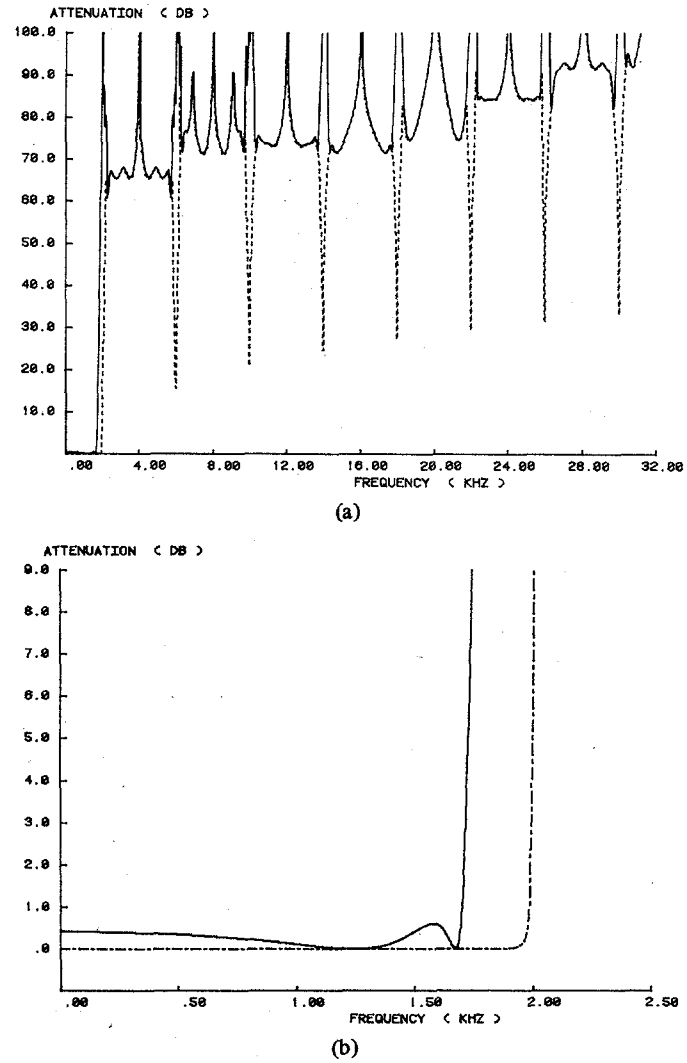

Fig. 8. Response of 60-channel transmultiplexer filter with three all-pass sections: —— = LP filter and all-pass sections, – – – = all-pass sections only. (a) Stopband response. (b) Passband response.

that the only real difference between the two cases is that the phase difference at $\omega = \pi$ is $(1 - 1/128)\pi$ instead of $(1 - 1/8)\pi$. Thus, a somewhat wider phase difference has to be approximated in the 60-channel case. In Fig. 7, the group delay is depicted.

If the number of all-pass sections is reduced to three, we will get the response of Fig. 8. The increase in attenuation towards higher frequencies can be clearly seen, and the effect of removing the LP filter is also demonstrated. The ripple in the passband and the stopband attenuation in the frequency band next to the passband are almost completely determined by the LP filter, as mentioned earlier. The resulting response without the LP filter might be sufficient for the CCITT requirements, but this question has not been thoroughly settled yet.

These filters can be realized with only 10 bits precision in fixed-point arithmetic. This implies, e.g., that if the signal requires a representation of 16 bits, a standard 16 × 16 bit multiplier could be used if the transmultiplexer is built around a central processing unit.

V. Conclusion

The design procedure presented is suitable for designing digital low-pass filters as polyphase networks in which the branch filters are realized by low-pass filters in cascade with all-pass sections. Since the starting point is not a prototype filter whose transfer function is to be synthesized, we cannot design filters with arbitrarily prescribed stopbands as in [2]. Due to this and the damping effects of (2.1), there is usually an excess of attenuation towards higher frequencies. This indicates that it should be possible to improve the design further with more sophisticated optimization techniques. An interesting problem in this context is to find out just what conditions must be fulfilled in order for the polyphase network to realize arbitrary low-pass transfer functions.

The main advantage with this method is the possibility of realizing the branch filters as WD filters, which gives a minimum amount of hardware, better coefficient sensitivities (and thus better noise properties), and a lower order of the networks involved. The success of the procedure is due to the fact that the low-pass filters are independently designed by standard WD design techniques, while the all-pass sections can be approximated in the z-domain and still be realized as WD filters. There still remain the problems of finding a direct synthesis method and of whether there are structures with even better properties for the polyphase network.

Appendix

A. The Transfer Function of $H(z)$

Consider the transfer functions $H(Z)$ and $H_m(z)$ and let the impulse responses be $\{h(n)\}$ and $\{h_m(n)\}$, respectively. Suppose these sequences are related by

$$h_m(n) = h(nN + m) \qquad n = 0, 1, \cdots \qquad m = 0, 1, \cdots, N - 1. \tag{A.1}$$

To express $H_m(z)$ in $H(Z)$, we begin by taking the z-transform of (A.1):

$$H_m(z) = \sum_n h_m(n) z^{-n} = \sum_n h(nN + m) z^{-n} = \{nN + m = k\}$$
$$= \sum_k c_k h(k) z^{-[(k-m)/N]}$$

where

$$c_k = \begin{cases} 1 & \text{if } k = nN + m \\ 0 & \text{otherwise.} \end{cases}$$

Now it is known [9] that

$$\frac{1}{N} \sum_{l=0}^{N-1} W_N^{-lr} = \begin{cases} 1 & \text{if } r = nN \\ 0 & \text{otherwise} \end{cases} \tag{A.2}$$

where

$$W_N = e^{-j(2\pi/N)}.$$

Putting $r = k - m$ gives

$$c_k = \frac{1}{N} \sum_{l=0}^{N-1} W_N^{-l(k-m)}$$
$$= \begin{cases} 1 & \text{if } (k - m) = nN, \quad k = nN + m \\ 0 & \text{otherwise.} \end{cases} \tag{A.3}$$

Thus, we have

$$H_m(z) = \sum_k h(k) \cdot \frac{1}{N} \sum_{l=0}^{N-1} W_N^{l(k-m)} \cdot z^{-[(k-m)/N]}$$
$$= \frac{1}{N} z^{m/N} \cdot \sum_{l=0}^{N-1} W_N^{lm} \cdot \sum_k h(k)(z^{1/N} W_N^l)^{-k}$$
$$= z^{m/N} \cdot \frac{1}{N} \sum_{l=0}^{N-1} W_N^{lm} H(z^{1/N} W_N^l). \tag{A.4}$$

To see that (A.4) realizes (2.1), we form the sum

$$\sum_{m=0}^{N-1} Z^{-m} H_m(Z^N) = \sum_{m=0}^{N-1} Z^{-m} (Z^N)^{m/N} \cdot \frac{1}{N} \sum_{l=0}^{N-1} W_N^{lm} H(ZW_N^l)$$
$$= \sum_{l=0}^{N-1} H(ZW_N^l) \cdot \frac{1}{N} \sum_{m=0}^{N-1} W_N^{lm}$$
$$= H(ZW_N^0) \cdot 1 + 0 = H(Z).$$

B. The Group Delay of $H(Z)$

Let $\varphi_m(\omega)$ be the phase response of $H_m(e^{j\omega})$. If we assume that there are no approximation errors, we have from (3.4)

$$\varphi_m(\omega) = \varphi_k(\omega) + \omega \frac{m - k}{N}.$$

If we use this in (2.1) and consider the baseband ($|\theta| \leq \pi/N$) only, we will get

$$H(e^{j\theta}) = \sum_{m=0}^{N-1} e^{-j\theta m} \cdot H_m(e^{j\theta N})$$
$$= \sum_{m=0}^{N-1} e^{-j\theta m} \cdot e^{j[\varphi_k(\theta N) + \theta(m-k)]}$$
$$= \sum_{m=0}^{N-1} e^{j[\varphi_k(\theta N) - \theta k]}.$$

Thus, $\varphi(\theta) = \varphi_k(\theta N) - \theta k = \varphi_o(\theta N)$, which means that the group delay of $H(Z)$ is approximately equal to that of $H_o(z)$.

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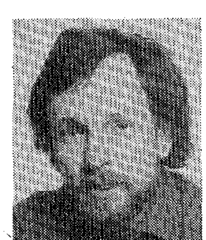

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